Review Handout For Math 1220

REFERENCE PAGES

TRIGONOMETRY

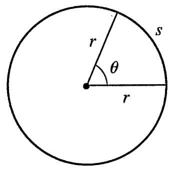
ANGLE MEASUREMENT

π radians $= 180°$

$1° = \dfrac{\pi}{180}$ rad $1 \text{ rad} = \dfrac{180°}{\pi}$

$s = r\theta$

(θ in radians)

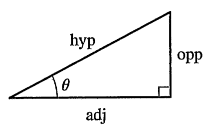


RIGHT ANGLE TRIGONOMETRY

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$

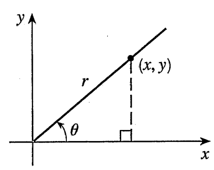


TRIGONOMETRIC FUNCTIONS

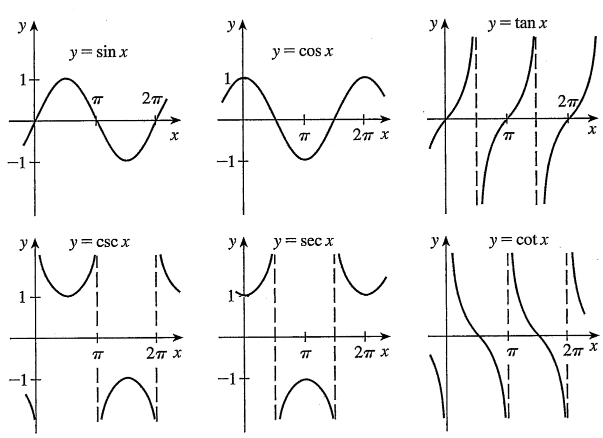
$\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$



GRAPHS OF TRIGONOMETRIC FUNCTIONS



TRIGONOMETRIC FUNCTIONS OF IMPORTANT ANGLES

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

FUNDAMENTAL IDENTITIES

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

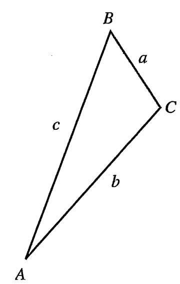
$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$ $\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$ $\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

THE LAW OF SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



THE LAW OF COSINES

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

ADDITION AND SUBTRACTION FORMULAS

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

DOUBLE-ANGLE FORMULAS

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

HALF-ANGLE FORMULAS

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$ $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

Logarithms

Suppose u and b are positive numbers and $b \neq 1$.

$\log_b u$ is the power of b which gives u: $p = \log_b u \Leftrightarrow u = b^p$.

Suppose u, v, a and b are positive numbers with $a \neq 1$ and $b \neq 1$, and p is any real number.

$\log u = \log_{10} u\,,\ \ln u = \log_e u$	$\log_a u = \frac{\log_b u}{\log_b a}$
$\log_b 1 = 0\,,\ \log_b b = 1$	$\log_b u = \log_b v \Longrightarrow u = v$
$\log_b u^p = p\,\log_b u$	$b^{\log_b u} = u$
$\log_b uv = \log_b u + \log_b v$	$\log_b \frac{u}{v} = \log_b u - \log_b v$

Inverse Trigonometric Functions

For $|x| \leq 1$, $\sin^{-1} x$ is the angle θ such that $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $\sin\theta = x$.

For $|x| \leq 1$, $\cos^{-1} x$ is the angle θ such that $0 \leq \theta \leq \pi$ and $\cos\theta = x$.

For any real number x, $\tan^{-1} x$ is the angle θ such that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $\tan\theta = x$.

For any real number x, $\cot^{-1} x$ is the angle θ such that $0 < \theta < \pi$ and $\cot\theta = x$.

For $|x| \geq 1$, $\sec^{-1} x$ is the angle θ such that $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$ and $\sec\theta = x$.

For $|x| \geq 1$, $\csc^{-1} x$ is the angle θ such that $0 < \theta \leq \frac{\pi}{2}$ or $\pi < \theta \leq \frac{3\pi}{2}$ and $\csc\theta = x$.

For $-1 \leq x \leq 1\,,\ \sin(\sin^{-1} x) = x$	For $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\,,\ \sin^{-1}(\sin x) = x$
For $-1 \leq x \leq 1\,,\ \cos(\cos^{-1} x) = x$	For $0 \leq x \leq \pi\,,\ \cos^{-1}(\cos x) = x$
For any real number x, $\tan(\tan^{-1} x) = x$	For $-\frac{\pi}{2} < x < \frac{\pi}{2}\,,\ \tan^{-1}(\tan x) = x$
For any real number x, $\cot(\cot^{-1} x) = x$	For $0 < x < \pi\,,\ \cot^{-1}(\cot x) = x$
For $x \leq -1$ or $x \geq 1$, $\sec(\sec^{-1} x) = x$	For $0 \leq x < \frac{\pi}{2}$ or $\pi \leq x < \frac{3\pi}{2}$, $\sec^{-1}(\sec x) = x$
For $x \leq -1$ or $x \geq 1$, $\csc(\csc^{-1} x) = x$	For $0 < x \leq \frac{\pi}{2}$ or $\pi < x \leq \frac{3\pi}{2}$, $\csc^{-1}(\csc x) = x$

Rectangular and Polar Coordinates

Rectangular Coordinate	Polar Coordinate
$P = (x,\, y) \Longrightarrow$	$P = (r,\, \theta)$ where $r = \sqrt{x^2 + y^2}$ and $\theta = \begin{cases} \tan^{-1}(\frac{y}{x}) & \text{, if } x > 0 \\ \pi + \tan^{-1}(\frac{y}{x}) & \text{, if } x < 0 \end{cases}$
$P = (x,\, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$	$\Longleftarrow P = (r,\, \theta)$

Series

$$\sum_{k=1}^{n} a_k = a_1 + \cdots + a_n$$

$$\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k \qquad \sum_{k=m}^{n} a_k = \sum_{k=m+i}^{n+i} a_{k-i}$$

$$\sum_{k=1}^{n} c = nc \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Arithmetic Series: $\sum_{k=1}^{n}[a + (k-1)d] = \frac{n}{2}[2a + (n-1)d]$

Finite Geometric Series: $\sum_{k=1}^{n} a\, r^{k-1} = \frac{a(1-r^n)}{1-r}$

Infinite Geometric Series: If $|r| < 1$, $\sum_{k=1}^{\infty} a\, r^{k-1} = \frac{a}{1-r}$

Differentiation Rules

$\frac{d}{dx}(c) = 0$ $\qquad$ $\frac{d}{dx}(x^n) = n\, x^{n-1}$

$\frac{d}{dx}[cf(x)] = cf'(x)$ $\qquad$ $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$ $\qquad$ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$

$\frac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\qquad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Fundamental Theorem of Calculus

For a continuous function f, $\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a)$ where $F'(x) = f(x)$

For a continuous function f, $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

Integration Formulas

$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	$\int cf(x)\,dx = c\int f(x)\,dx$
$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$	
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$

Strategies For Integration

Method	Example
Expanding	$(x - x^{-1})^2 = x^2 - 2 + x^{-2}$
Using a trigonometric identity	$\sin^2 x = \frac{1}{2}[1 - \cos(2x)]$
Simplifying a fraction	$\frac{5x^6 + 3}{x^2} = 5x^4 + 3x^{-2}$
Substitution: For $u = g(x)$,	For $u = x^2 + 1$ we have $x\,dx = \frac{du}{2}$
$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$	and so $\int \frac{x}{\sqrt{x^2+1}}\,dx = \int \frac{du}{2\sqrt{u}}$

For application of integrals, such as finding areas and volumes, see Chapter 5 of your textbook.